Stable Upright Walking and Running using a simple Pendulum based Control Scheme

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One of the major issues in humanoid walking and running is to keep the trunk upright while the system is basically an unstable inverted pendulum. Here, we investigate trunk stability based on the bipedal spring-loaded inverted pendulum (SLIP) model. The proposed control strategy is to redirect the ground reaction force (GRF) to a point on the trunk located above the center of mass.

For keeping the trunk upright, no external sensors are required. In a perturbed situation, the proposed strategy leads to pendulum-like pitching motions. The model predicts a hip torque similar in shape and magnitude to that observed in human walking.

Keywords: SLIP; postural stability; trunk; hip torque; Virtual Pivot Point.

1. Introduction

The natural gaits of human locomotion are walking and running. While the legs are very complex mechanical systems, their dynamics in locomotion are well described by spring-like functions. The simplest model describing both gaits in a self-stable manner is the bipedal spring-mass model[1] which is an extension of the well known Spring Loaded Inverted Pendulum (SLIP).[2] One of the strong simplifications is the representation of the body by a point mass. Hence, trunk stabilization is not addressed. Anyhow, this is a major problem in bipedal robots.

In robots with spring-like leg behavior a common strategy for keeping the trunk upright is to measure the pitch angle with respect to the ground and to apply a PD control[3,4] or a higher level control.[5]

In this study we propose a novel strategy to stabilize the trunk. Here, we apply a hip torque such that the ground reaction force (GRF) directs to a point on the body axis above the center of mass (COM). This concept

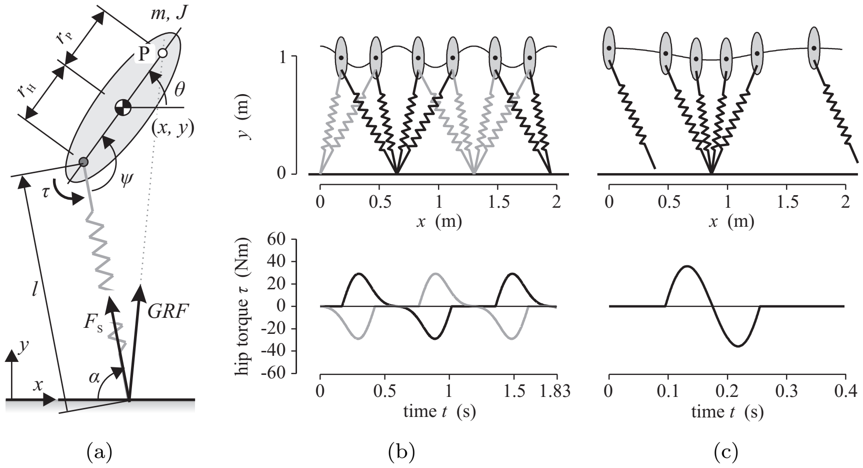


Fig. 1. (a) Model with spring legs and rigid trunk. Note that the model has two legs while only one is shown. Periodic sequences of (b) walking and (c) running with corresponding hip torques.

leads to a damped pendulum-like pitch motion during walking and running.

2. Methods

The model we use here is a bipedal SLIP with a rigid trunk (Fig. 1(a)) similar to the Asymmetric Spring Loaded Inverted Pendulum (ASLIP).[5] The legs are represented by massless springs with leg stiffness k generating a leg force F_S. The trunk is a rigid body with mass m, moment of inertia J and a COM located above the hip. The values are shown in Tab. 1.

The strategy for stabilizing the trunk is to direct the GRF of a leg

Table 1. Model parameters

parameter	symbol	value
body mass	m	80 kg
inertia	J	4.58 kg m^2
initial leg length	l_0	1 m
leg stiffness	k	20 kN m^{-1}
distance hip - COM	r_H	0.1 m
distance COM - P	r_P	0.1 m
landing angle	α_0	67 deg

to a certain point P located at the body axis[a] as illustrated in Fig. 1(a). Herefore, we apply a hip torque τ for each leg as exemplarily given for the right leg (the additional index R denotes the right leg):

$$\tau_R = F_{S,R}\, l_R\, \frac{(r_H + r_P)\sin\psi_R}{l_R - (r_H + r_P)\cos\psi_R}. \tag{1}$$

Positive values correspond to a hip extension. The GRF for the right leg is

$$\boldsymbol{GRF_R} = \boldsymbol{F_{S,R}} + \frac{\tau_R}{l_R}\begin{pmatrix}\sin\alpha_R\\ \cos\alpha_R\end{pmatrix}. \tag{2}$$

GRF and hip torque are zero during swing phase. We consider transitions between swing and stance by the following conditions: swing → stance: $y(t) = l_0 \sin\alpha_0$ with $\dot{y} < 0$ and stance → swing: $l(t) = l_0$ with $\dot{l} > 0$. The parameters l_0 and α_0 are initial leg length and landing angle, respectively.

We analyze dynamic stability of the system using a Poincaré map $\boldsymbol{F}$ for the system variables (y, $\dot{x}$, θ, $\dot{\theta}$) at the instant of upper apex. The system is stable if (1) a periodic solution exists (subsequent apices are equal) and (2) all eigenvalues of the Jacobian of $\boldsymbol{F}$ at the periodic solution have magnitudes less than one. We use a Newton-Raphson algorithm to find periodic solutions.

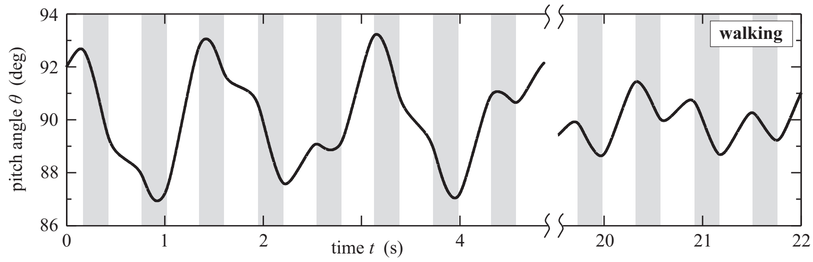
3. Results

Periodic solutions exist for both gaits, walking and running (Fig. 1(b) and 1(c)), with small pitching motions. The strategy for generating the hip torque is equal in both gaits. Fig. 1(b) shows that in walking the hip torque is almost zero during midstance.

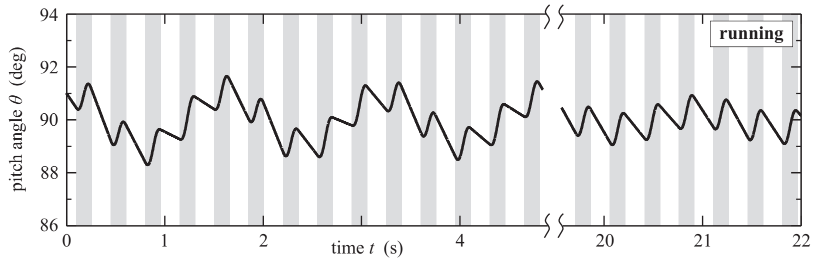
For the selected solutions, three out of four eigenvalues of the Jacobian of $\boldsymbol{F}$ lie within the unit circle and one eigenvalue is one. Once these patterns are perturbed the system tends to another periodic solution. Such periodic patterns are called partially asymptotically stable.[6] For every velocity within a certain range exists at least one periodic solution.

Fig. 2 shows the pitch angle of perturbed gait patterns. A body oscillation around the vertical axis occurs with a frequency similar to a pendulum (with moment of inertia J) which is mounted at the point P. The oscillation is slightly damped. Table 2 shows that the model can handle relatively large disturbances in running and comparatively small disturbances in walking.

[a]The body axis is defined as the connecting line between hip and COM.



(a)



(b)

Fig. 2. Pitch angle in perturbed walking (a), and running (b). The periodic patterns of Table 2 were perturbed by an increased initial pitch angle: 2 deg in (a) and 1 deg (b). Gray regions indicate stance phase in running and double support phase in walking.

4. Discussion and Conclusion

4.1. *General Discussion*

In this paper, an intuitive strategy for stabilizing the trunk in bipedal walking and running is presented. Except for the landing angle, this controller requires only internal sensors. The model does not preserve energy, however, it reveals periodic solutions which do not affect the system energy. Because the body oscillates like a pendulum mounted at the point P, we call P a *Virtual Pivot Point* (VPP). The emergence of stability can be understood as results of the underlying systems which are asymptotically stable (SLIP)[7] or indifferently stable (pendulum), respectively. Apparently the combination is such that the indifferent stable pendulum becomes asymtotically stable. The VPP-strategy could be a basic framework for investigating postural stability and the hip function in dynamic bipedal locomotion. Interestingly, the model predicts hip torque patterns for walking which are similar in shape and magnitude to corresponding data observed in human overground walking.[8]

Table 2. Initial conditions of the selected periodic solutions (middle column) and the range of initial conditions that lead to stable gaits.

initial	**walking**			**running**		
conditions	min	periodic	max	min	periodic	max
y_0 (m)	1.059	1.082	1.100	1.021	1.065	1.538
$\dot{x}_0$ (m/s)	1.021	1.038	1.076	4.81	5.00	5.93
θ_0 (deg)	87.3	90.0	93.3	46.6	90.0	132.1
$\dot{\theta}_0$ (deg/s)	-5.83	6.47	13.84	-202.2	-5.5	75.8

The proposed VPP-strategy offers an alternative concept to the ZMP[9,10] for stabilizing upright gait. In contrast to state-of-the-art humanoid robots, this control offers the unique opportunity to stabilize walking and running without relying on a specific foot shape. Here, gait stability does not depend on the size of the foot but largely on the maximum stiction force allowed by the ground-foot-contact. Assuming a stiction coefficient of 0.8, to achieve the same effect of a VPP in a ZMP-controlled biped the foot length would need to be over 1.6 times leg length. Therefore, both strategies could be used dependent on external conditions and geometry of the leg. For example, on slippery surfaces the VPP torque is limited whereas on uneven ground a ZMP-based robot might fail.

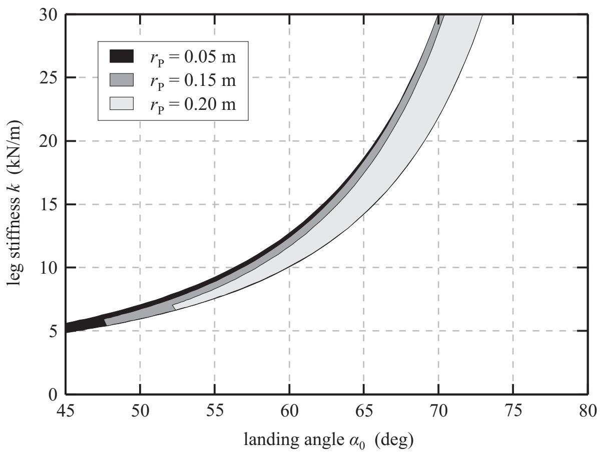


Fig. 3. Regions of (k, α) for stable running for a given speed ($\dot{x}$ = 5 m/s) and three positions of P. Only the left borders differ dependent on r_P.

4.2. *Comparison with the SLIP*

Stable running patterns exist within a region of parameter combinations (Fig. 3). This is a subset of the region of stable running in the SLIP model.[7] With increasing r_P, the size of the stable region is reduced.

The model is indifferently stable for $r_P = 0$ as the total torque vanishes and so any rotation will persist. For $r_P = r_H = 0$, the model is identical to the SLIP. There exists a minimum r_P for stable solutions which is approximately 0.01 m.

By shifting the VPP forward or backward with respect to the body axis, the hip torque pattern is changed such that the system energy is continuously decreasing or increasing, respectively. This additional breaking or thrusting force can be used to cope with external loads (e.g. carrying a cart or uphill locomotion). The corresponding adaptation of trunk posture is in line with the observations in human locomotion (e.g. trunk lent forward for acceleration) as well as to the concept of the Segway two-wheeled mobile systems[11] or similar robots.[12]

4.3. *Future Work*

We plan to implement the proposed VPP-strategy in various bipedal robots. Also, we plan to extend the VPP-strategy for three-dimensional locomotion.

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